Vetores

Tamanho

direção

sentido

início

Belavitis ⇒ Principio de Equipolência

sentido

Tamanho

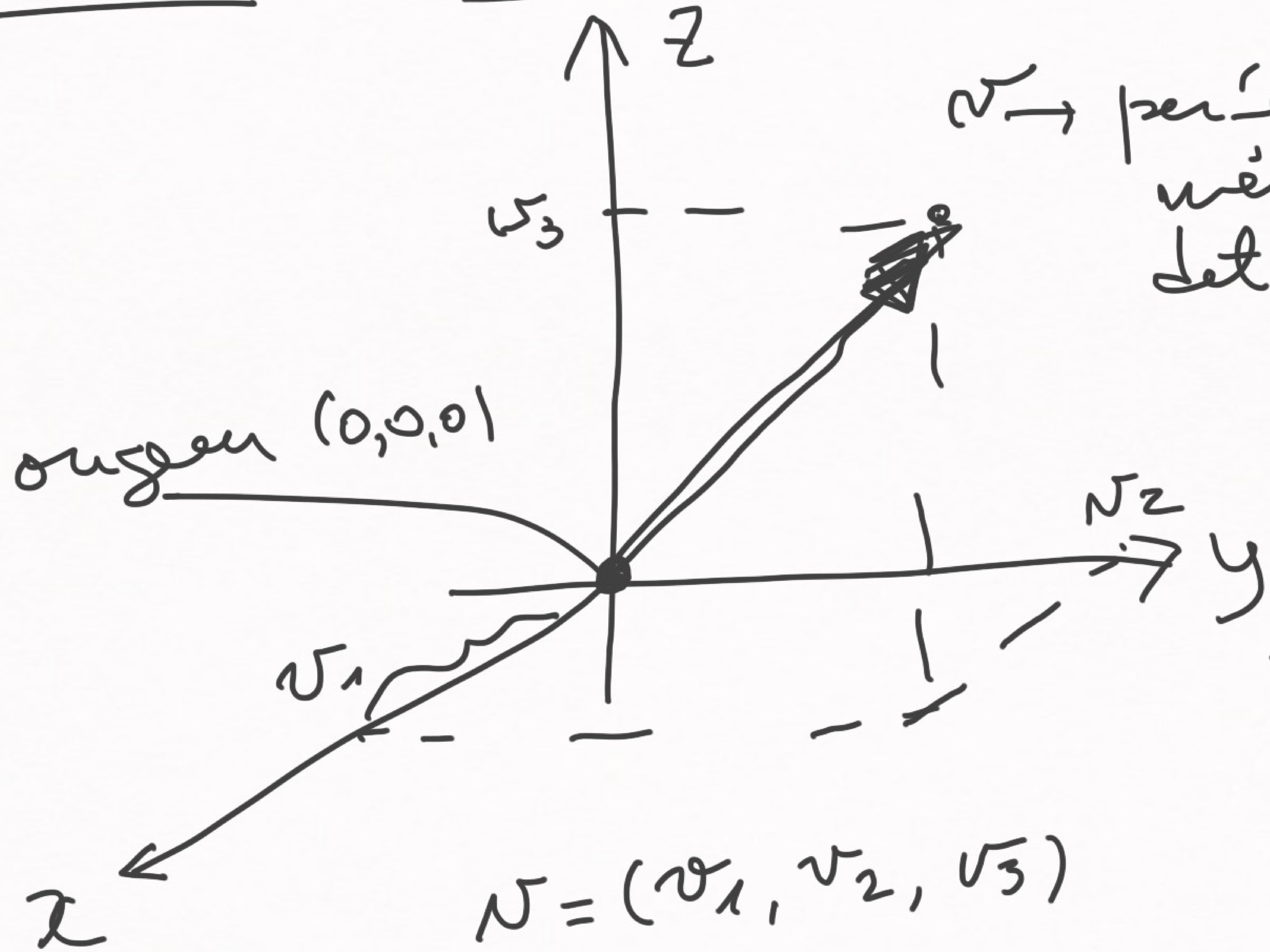
Equivalentes

mesmo tamanho

direção mesma

iguais

Coordenadas Cartesianas $\mathbb{R}^3$



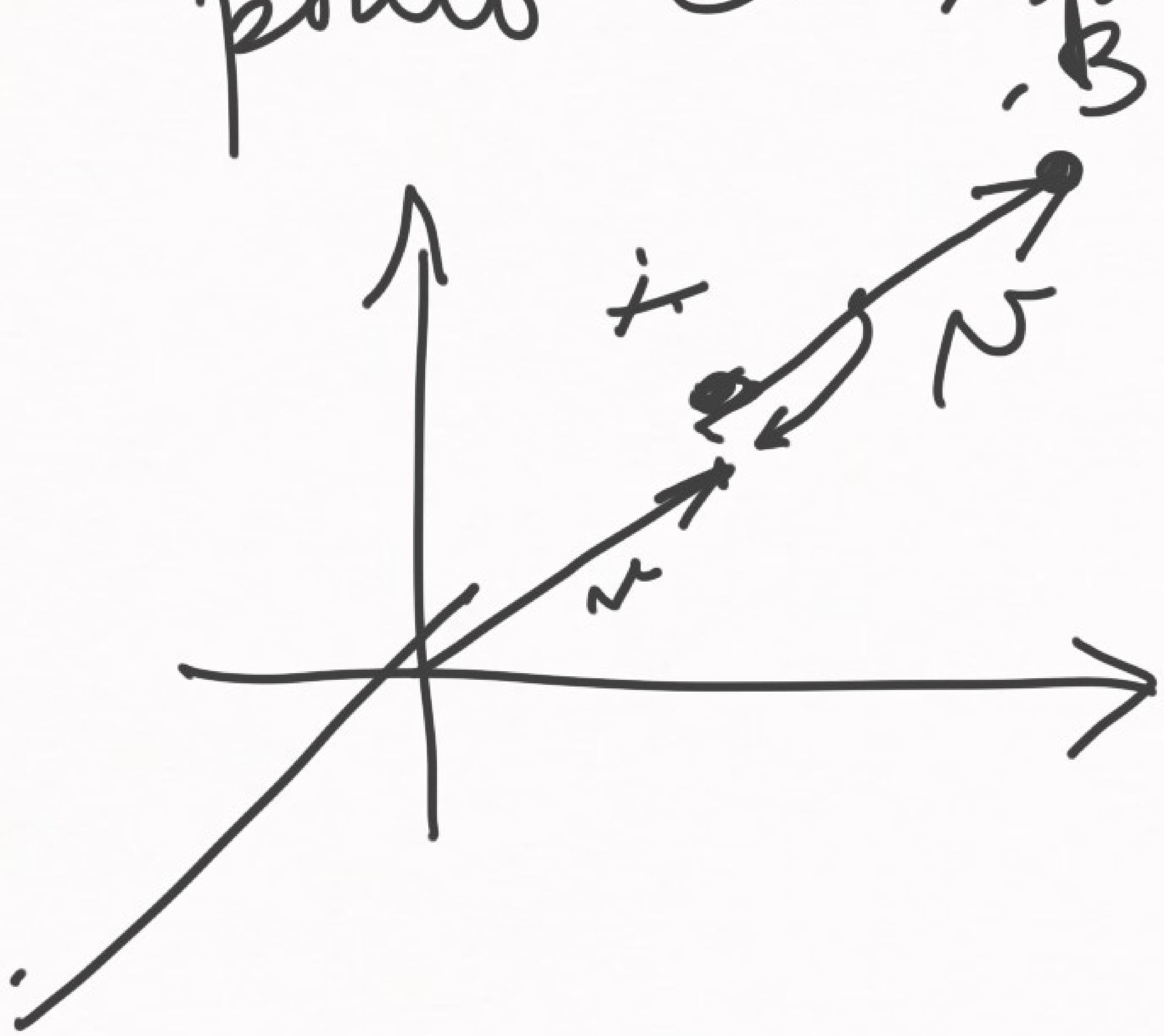
$v \rightarrow$ perfectamente determinado

$v = (v_1, v_2, v_3)$

$\mathbb{R}^2$ idem

Relação biunívoca entre os pontos do espaço e os vetores

cada ponto do espaço representa um vetor e cada vetor representa um ponto do espaço



$$v = \vec{AB}$$

$$v = (b_1, b_2, b_3) - (a_1, a_2, a_3)$$
$$= (b_1 - a_1, b_2 - a_2, b_3 - a_3)$$

$v = (v_1, v_2, v_3) \Rightarrow$ Tamanho

Magnitude

modulo

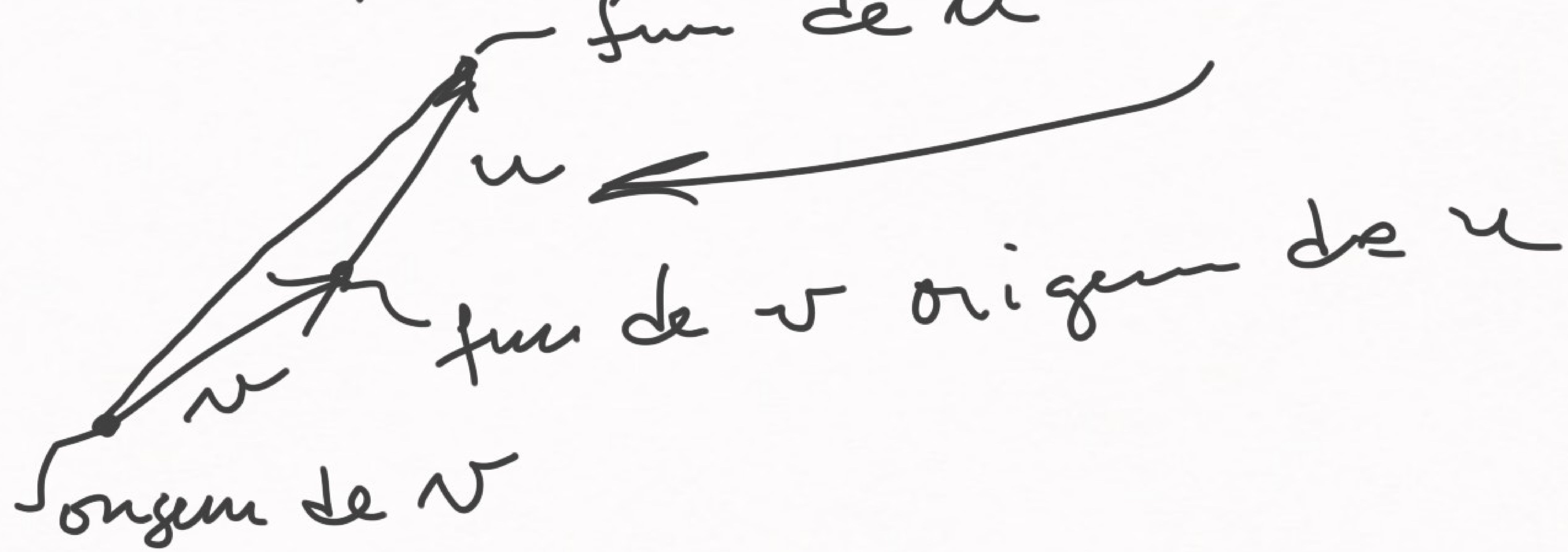
$\|v\| = \sqrt{v_1^2 + v_2^2 + v_3^2}$ → Pitagoras

tamanho

numero > 0

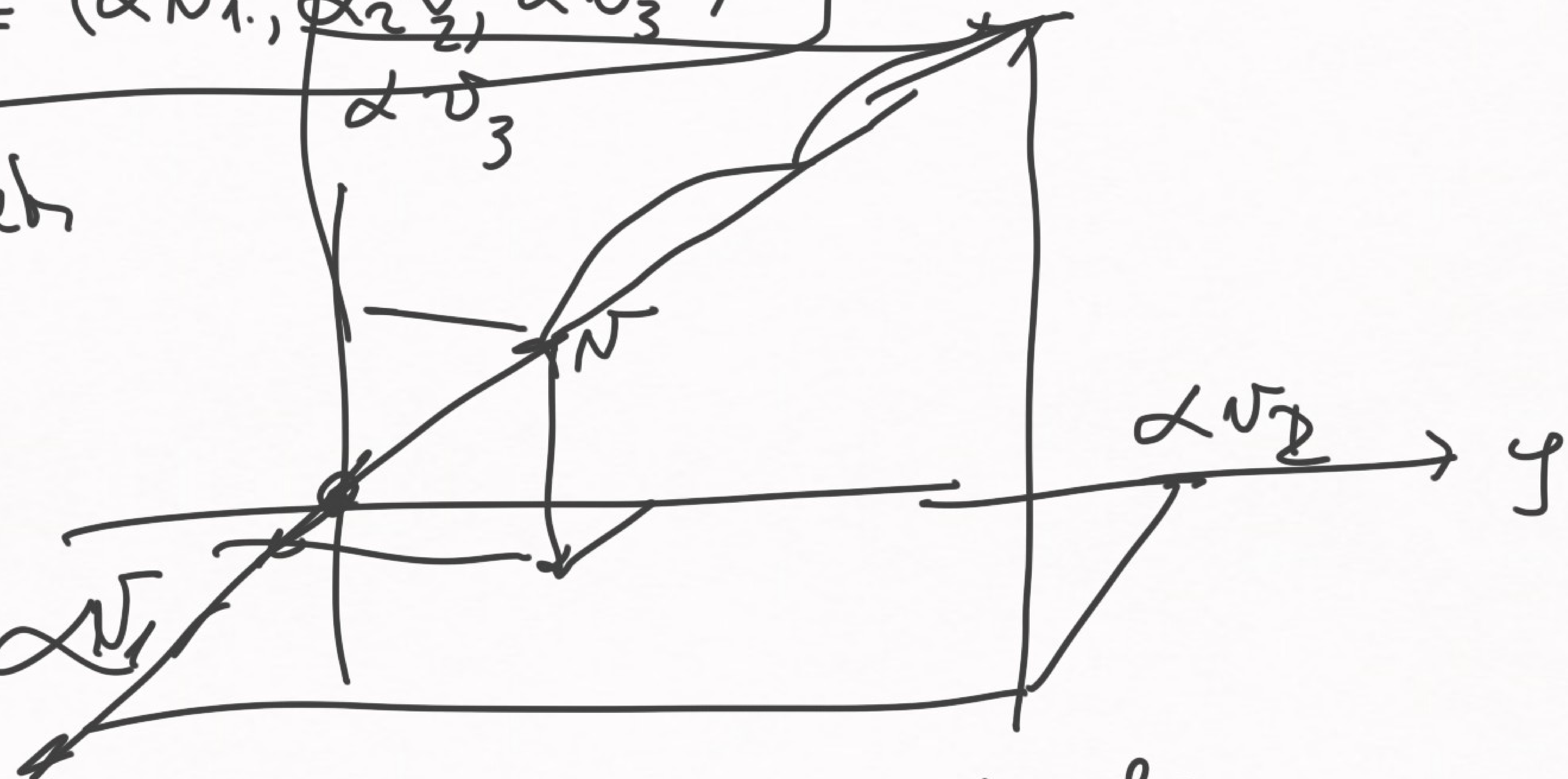
operações entre vetores

$v + u = (v_1 + u_1, v_2 + u_2, v_3 + u_3)$



$$\boxed{\alpha v = (\alpha v_1, \alpha v_2, \alpha v_3)}$$

$\in \mathbb{R}$ vet



$\alpha > 1 \rightarrow$ aumenta a magnitude

$\alpha < 0 \Rightarrow$ inverte o sentido

$0 < \alpha < 1 \Rightarrow$ diminui a magnitude

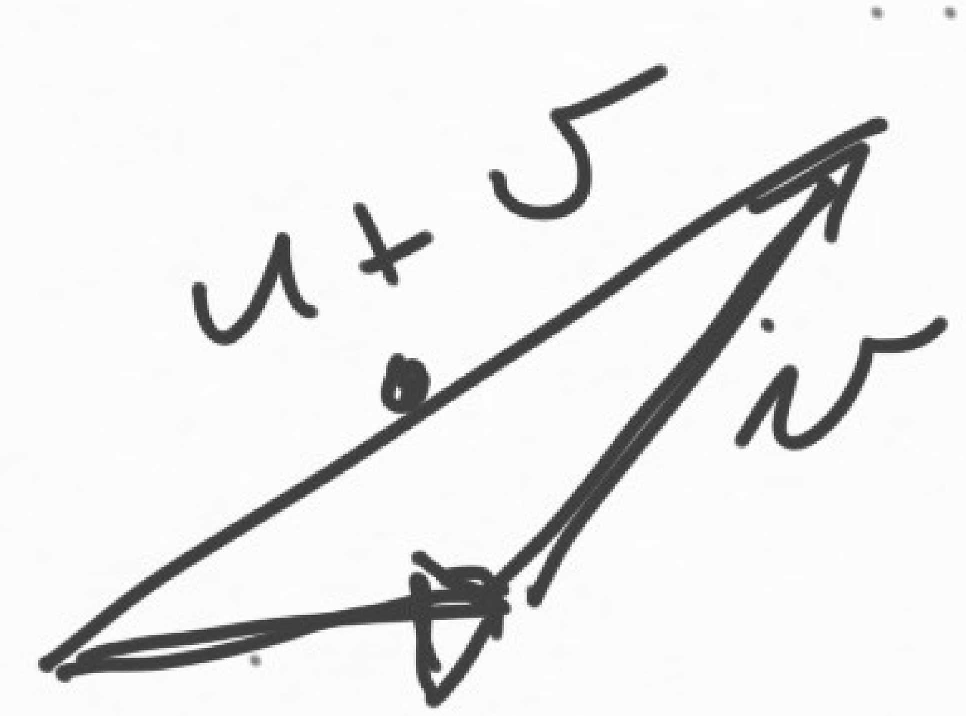
$$u - v = u + (-1)\, v = (u_1 - v_1,\ u_2 - u_2,\ u_3 - v_3)$$

$$\|u + v\| = \sqrt{(u_1 + v_1)^2 + (u_2 + v_2)^2 + (u_3 + v_3)^2}$$

$$\|u\| = 0 \implies u = 0$$

$$\|\alpha u\| = |\alpha|\, \|u\|$$

$$\|u + v\| \leq \|u\| + \|v\|$$



$$\left.\begin{aligned} \alpha(u+v) &= \alpha u + \alpha v \\ (\alpha+\beta)u &= \alpha u + \beta u \end{aligned}\right\}$$ dist.

$u + v = v + u$ com.

$u + v + w = u + (v + w)$ asso.

$\exists$ o <u>neutro</u> do soma

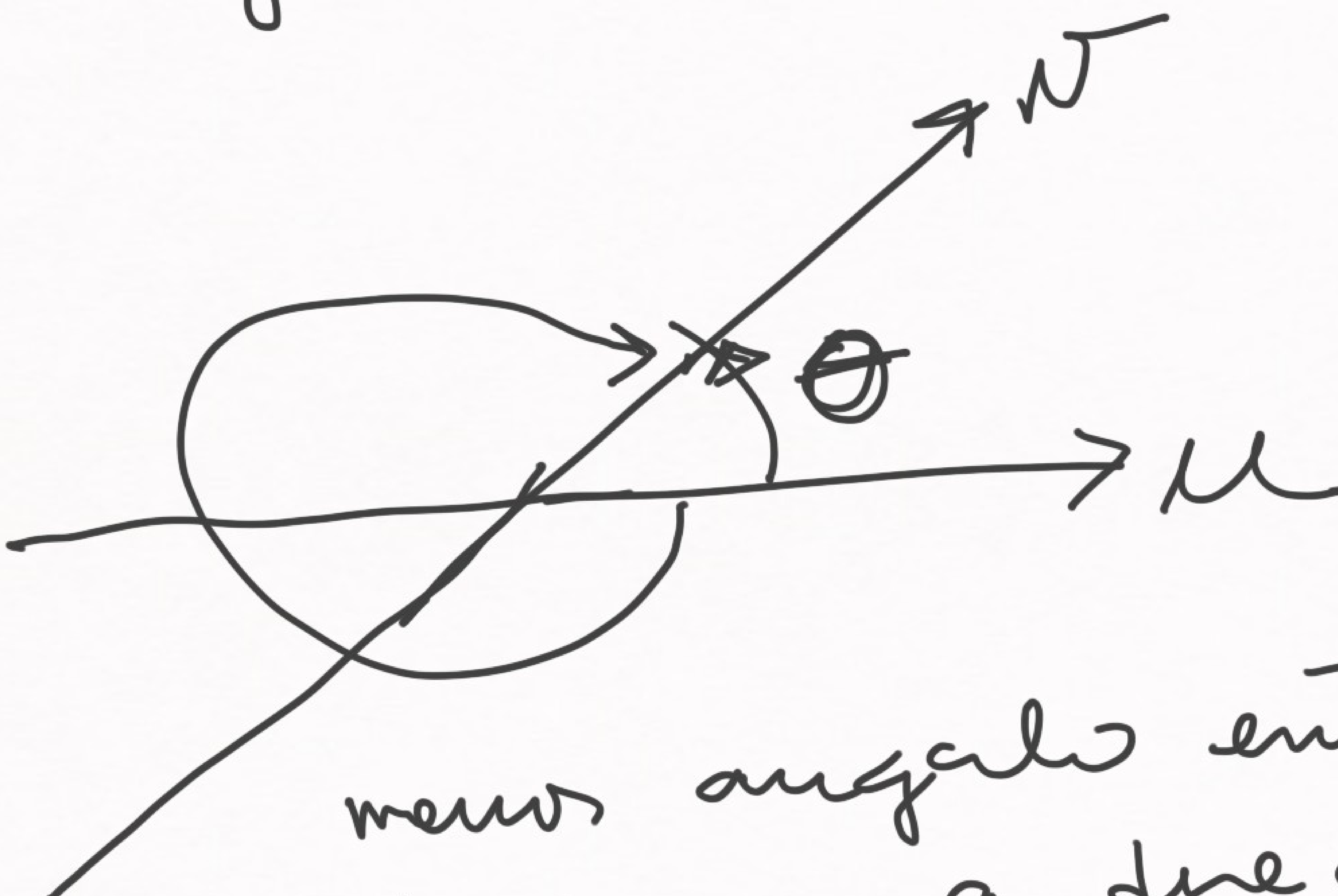
$u + 0 = 0 + u = u$

$\exists$ inverso aditivo $-u$

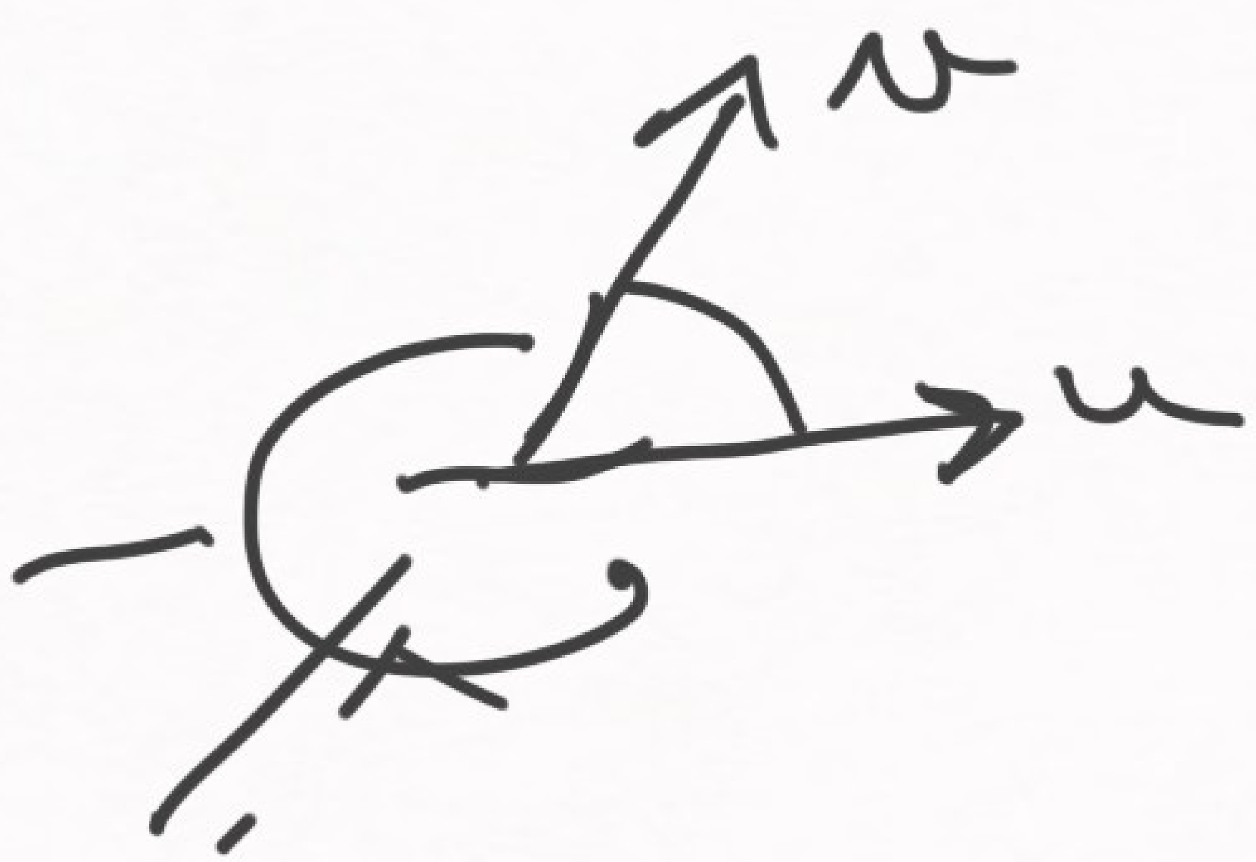
$u + (-u) = 0 \longrightarrow$ neutro

$\alpha = 1$ tq $\alpha u = u$ neutro. produto por escalar

Angulo entre vetores



menor angulo entre a direção de v e a direção de u levando o sentido



Produto entre vetores

$u \bullet v = \langle u, v \rangle \longrightarrow$ numero real

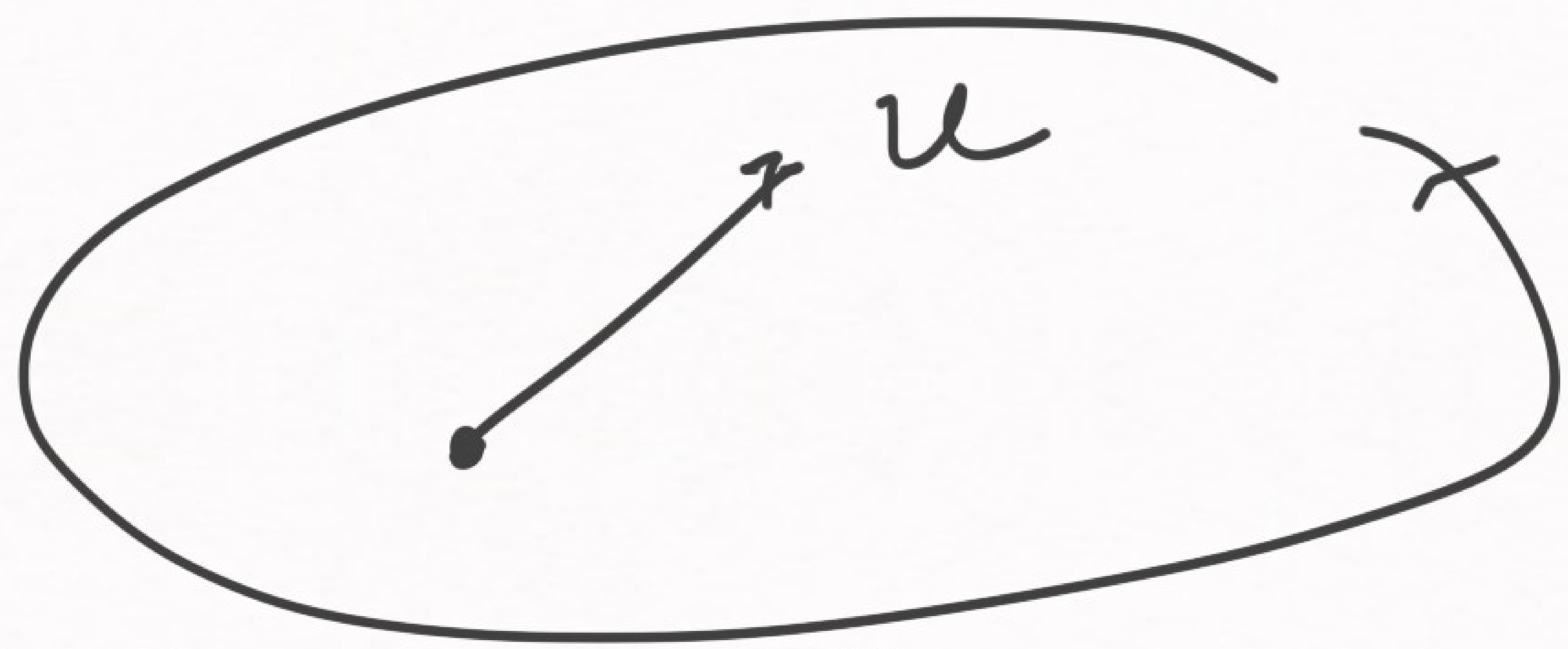
<u>escalar</u>

$\langle \; \rangle : \mathbb{R}^3 \times \mathbb{R}^3 \longrightarrow \mathbb{R}$

$\underline{\underline{u \times v}} = w \longrightarrow$ vector

$\times : \mathbb{R}^3 \times \mathbb{R}^3 \longrightarrow \mathbb{R}$ vectorial

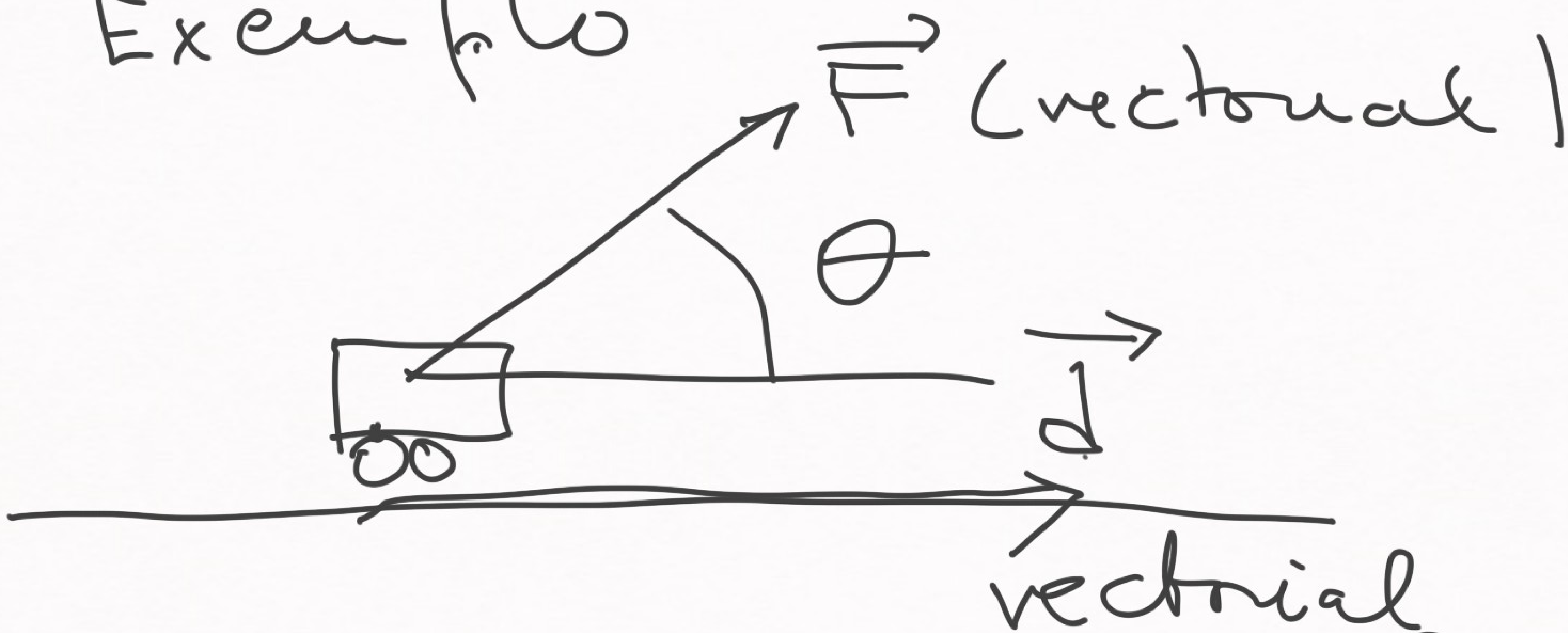
1°) no esta definido o angulo de u com o vetor nulo



Producto Escolar

Produto de dous magnitudes vectoriais resulte em uma magnitude escolar.

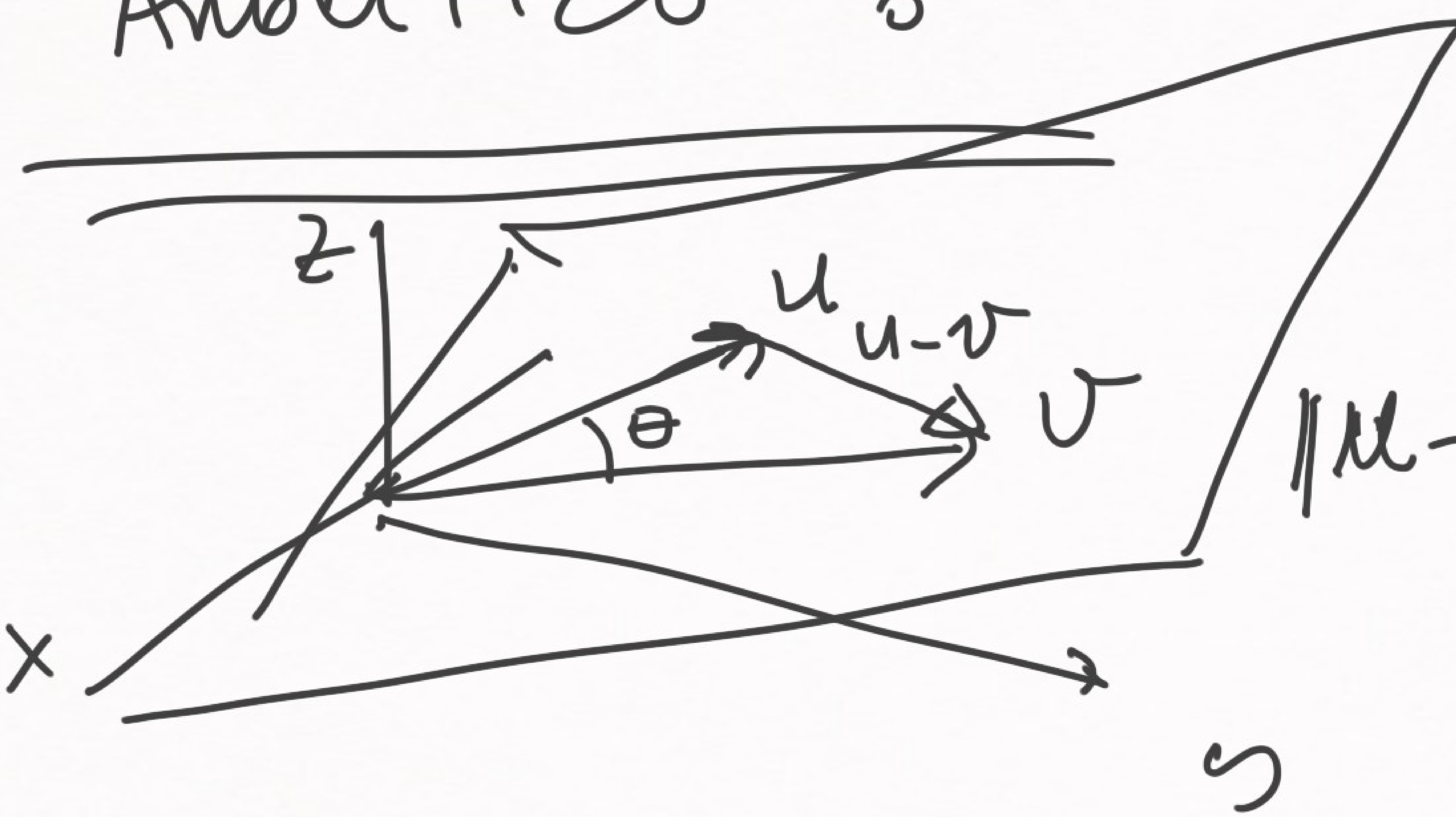
Exemplo



$$\vec{F} \cdot \vec{d} = \underbrace{\|F\| \, \|d\| \cos\theta}_{\text{energia.}}$$

$$u \cdot v = \langle u, v \rangle = \begin{cases} \|u\|\,\|v\| \cos \widehat{uv} & \text{se } u \neq 0 \text{ e } v \neq 0 \\ 0 & \text{caso contrário} \end{cases}$$

Analítico ?



Geometria

Teorema do coseno

$$\|u - v\|^2 = \|u\|^2 + \|v\|^2 - 2\|u\|\|v\| \cos\theta$$

//

$$\|u-v\|^2 = (u_1-v_1)^2+(u_2-v_2)^2+(u_3-v_3)^2 =$$

$$\cancel{u_1^2+u_2^2+u_3^2+v_1^2+v_2^2+v_3^2} - 2u_1v_1 - 2u_2v_2$$

$$-2u_3v_3$$

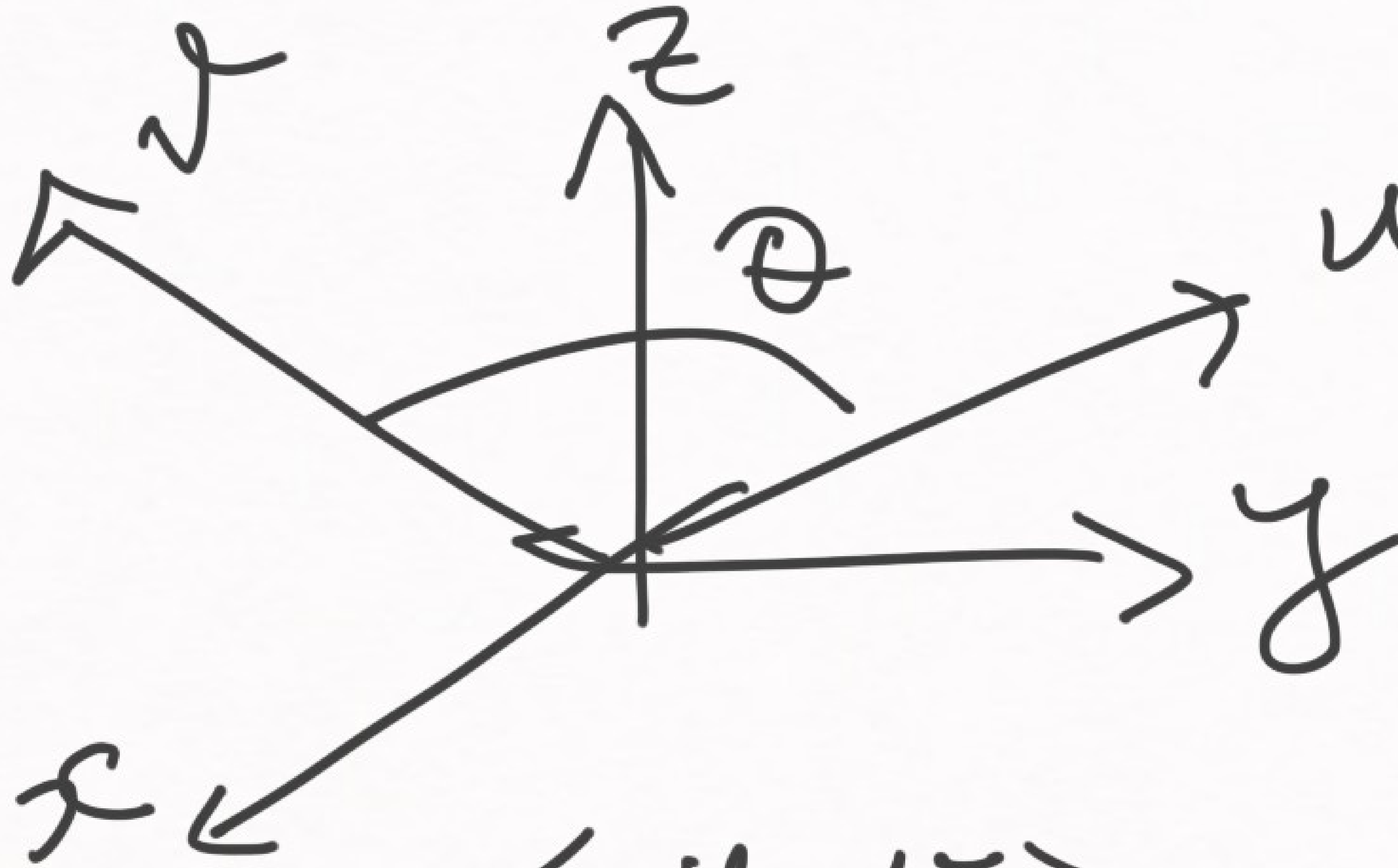
$$\|u\|^2+\|v\|^2-2\|u\|\,\|v\|\cos\theta = \cancel{u_1^2+u_2^2+u_3^2+}$$

$$\cancel{v_1^2+v_2^2+v_3^2} - 2\,\|u\|\,\|v\|\cos\theta$$

$$\cancel{-2}\,(u_1v_1+u_2v_2+u_3v_3) = \cancel{-2}\,\underbrace{\|u\|\cdot\|v\|\cos\theta}_{\langle u,v\rangle}$$

$$=$$

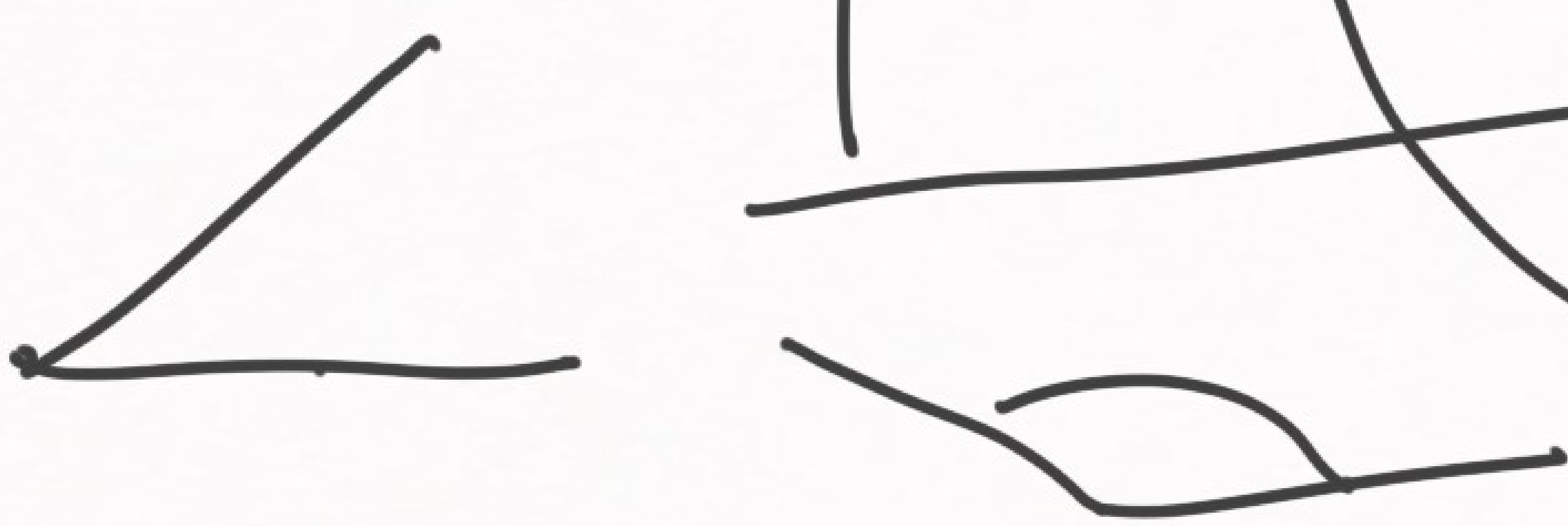
$$\langle u,v\rangle = u_1v_1+u_2v_2+u_3v_3$$



angulo de maneira analitica

$$\langle u, v \rangle = u_1 v_1 + u_2 v_2 + u_3 v_3 = \|u\| \, \|v\| \cos\theta$$

$$\boxed{\cos\theta = \frac{u_1 v_1 + u_2 v_2 + u_3 v_3}{\|u\| \, \|v\|}}$$



1° ou 2°

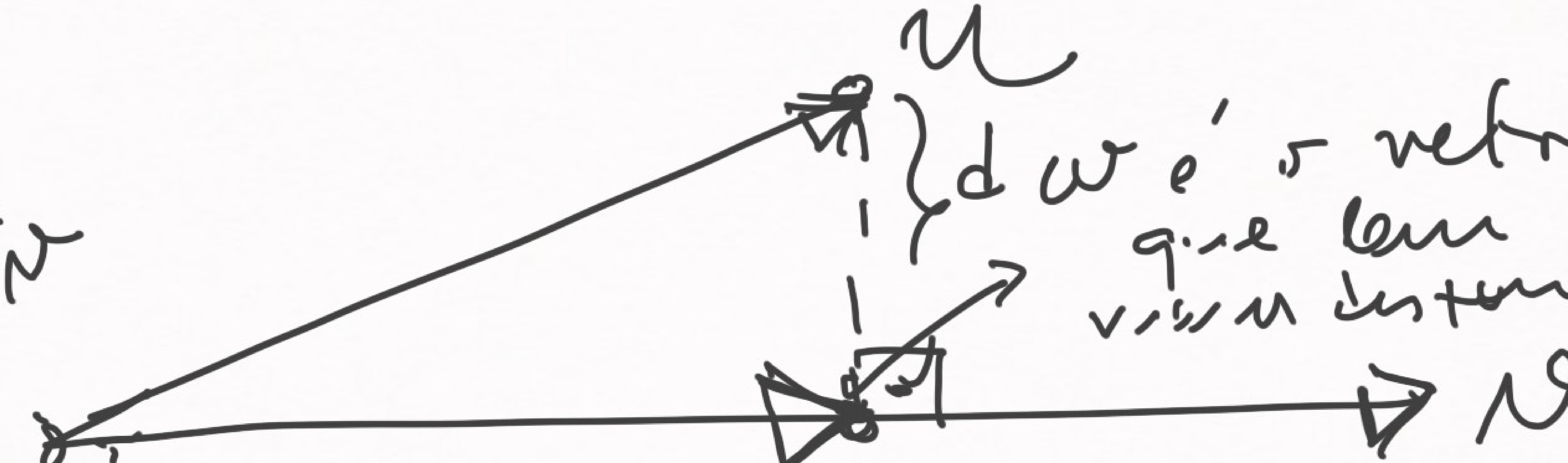
$u \perp v \iff \langle u, v \rangle = 0$

ortogonal a v

Caracteriza a ortogonalidade

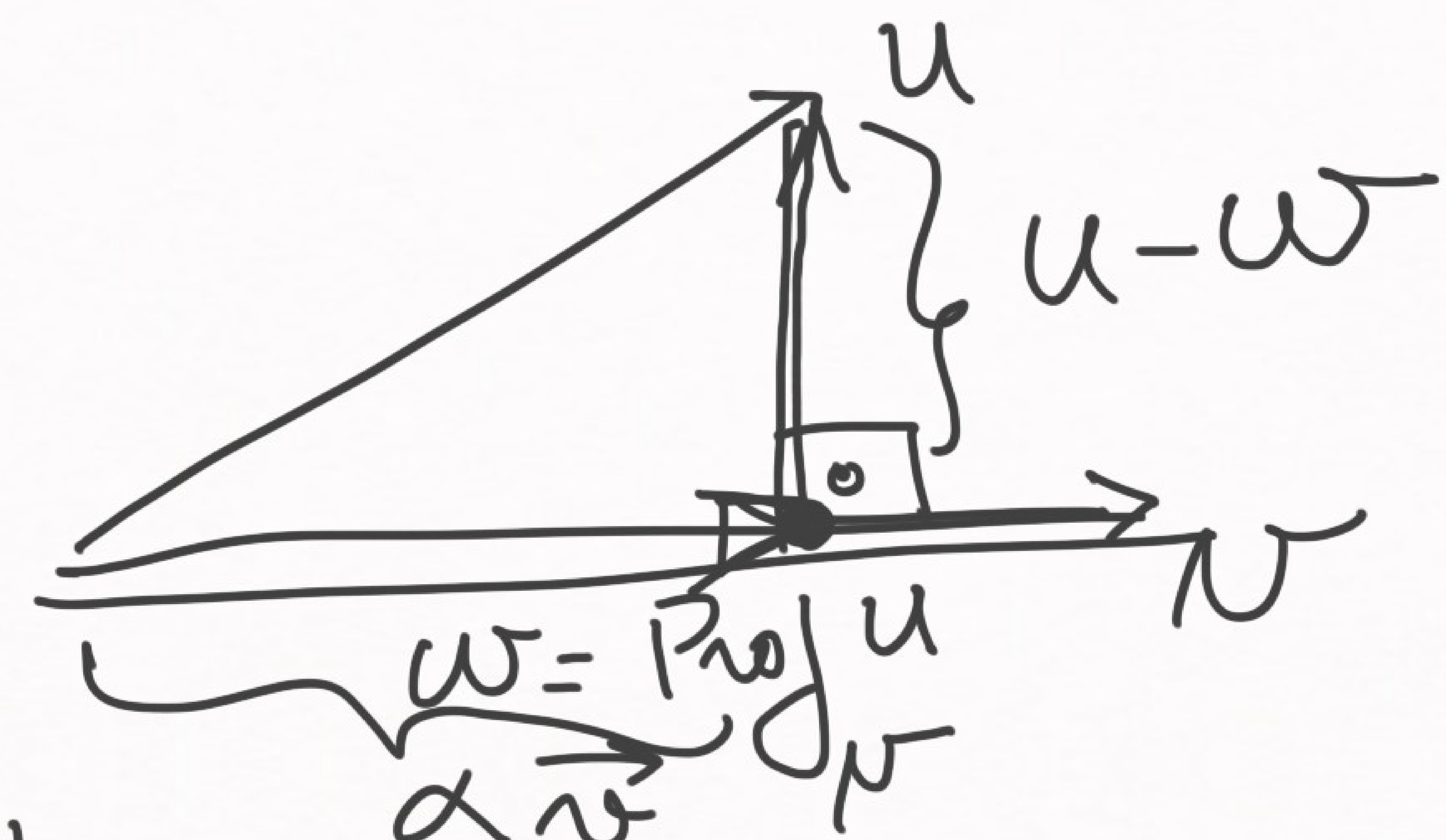
Projeção ortogonal

$v = \mathrm{Proj}_v^u$



w é o vetor que tem a mínima distância a u e está na reta de direção v

$w = \mathrm{Proj}_v^u$



$$(u-w) \perp v$$

$$\langle u-w, v\rangle = 0 \Rightarrow \underbrace{\langle u, v\rangle = \langle w, \vec{v}\rangle}$$

$$\mathrm{Proj}_v u = \alpha v \qquad \langle u, v\rangle = \langle \alpha\vec{v}, \vec{v}\rangle$$

$$\Rightarrow \alpha = \frac{\langle u, v\rangle}{\langle v, v\rangle}$$

$$\boxed{w = \mathrm{Proj}_v u = \frac{\langle u, v\rangle}{\langle v, v\rangle} \vec{v}}$$ ✓

$$\langle v, v\rangle = v_1 v_1 + v_2 v_2 + v_3 v_3 = \|v\|^2$$

$$= \frac{\langle u, v\rangle}{\|v\|} \frac{\vec{v}}{\|v\|}$$

reta de norma 1 $\Rightarrow \dfrac{\vec{v}}{\|v\|} \rightsquigarrow$ versor na direção v

$$v = v_1 e_1 + v_2 e_2 + v_3 e_3$$

$$\mu = u_1 e_1 + u_2 e_2 + u_3 e_3$$

$$e_1 e_2 = e_1 e_3 = 0 \qquad e_2 e_3 = 0$$

$$\perp \qquad \perp$$

$$\langle u, v \rangle = \langle v_1 e_1 + v_2 e_2 + v_3 e_3, u_1 e_1 + u_2 e_2 + u_3 e_3 \rangle$$

$$= v_1 \cancel{u_1} + v_2 u_2 + v_3 u_3$$

onde reale

commutativa

$e_1 e_1 = 1$, $e_1 e_2 = 0$, $e_2 e_3 = 0$

$e_2 e_2 = 1$, $e_3 e_3 = 1$

$$\cos\theta = \frac{\langle u \cdot v \rangle}{\|u\|\|v\|}$$

Produto vetorial

$$u \times v = w$$

↑ ↑ ↑ vetor

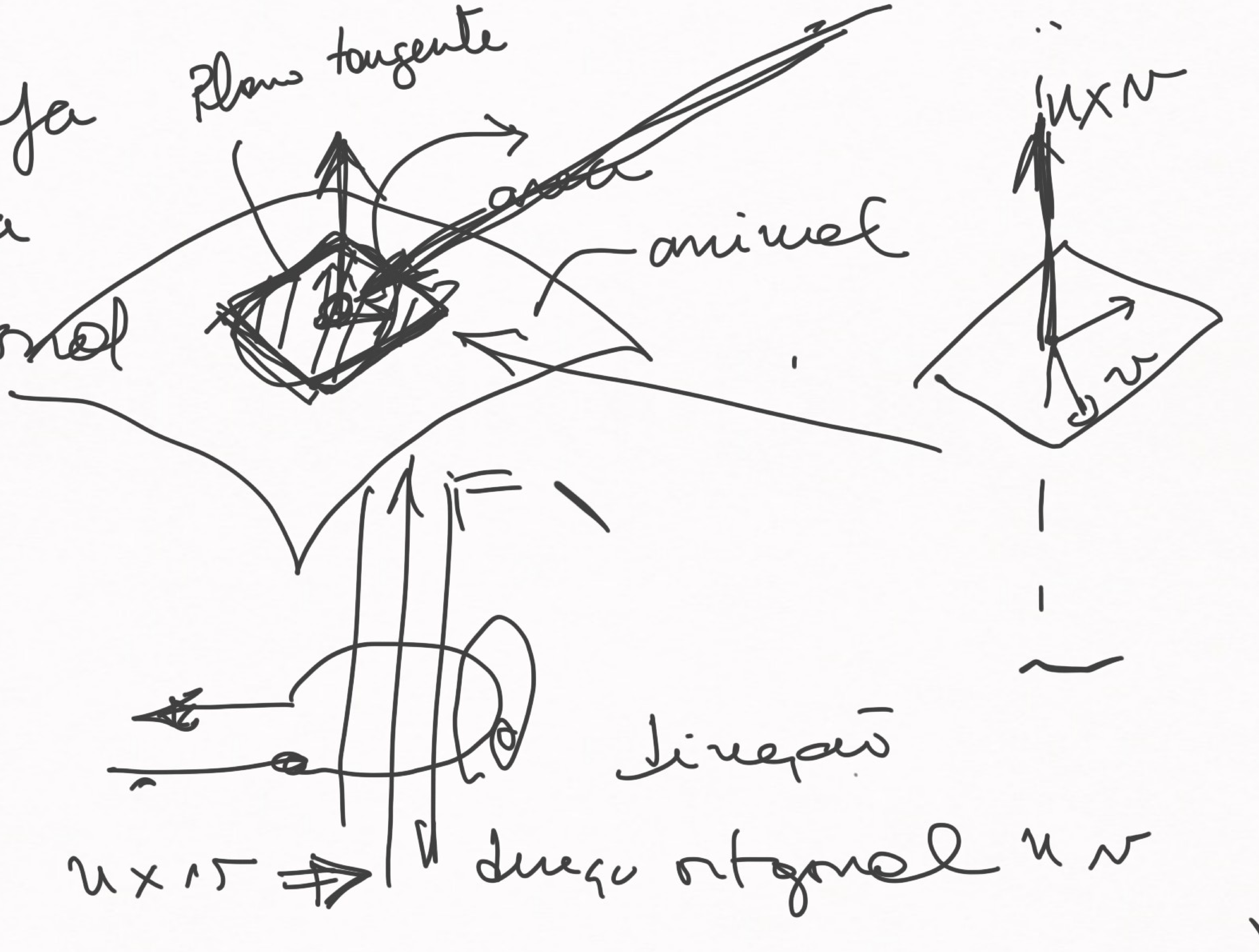
1_ magnitude

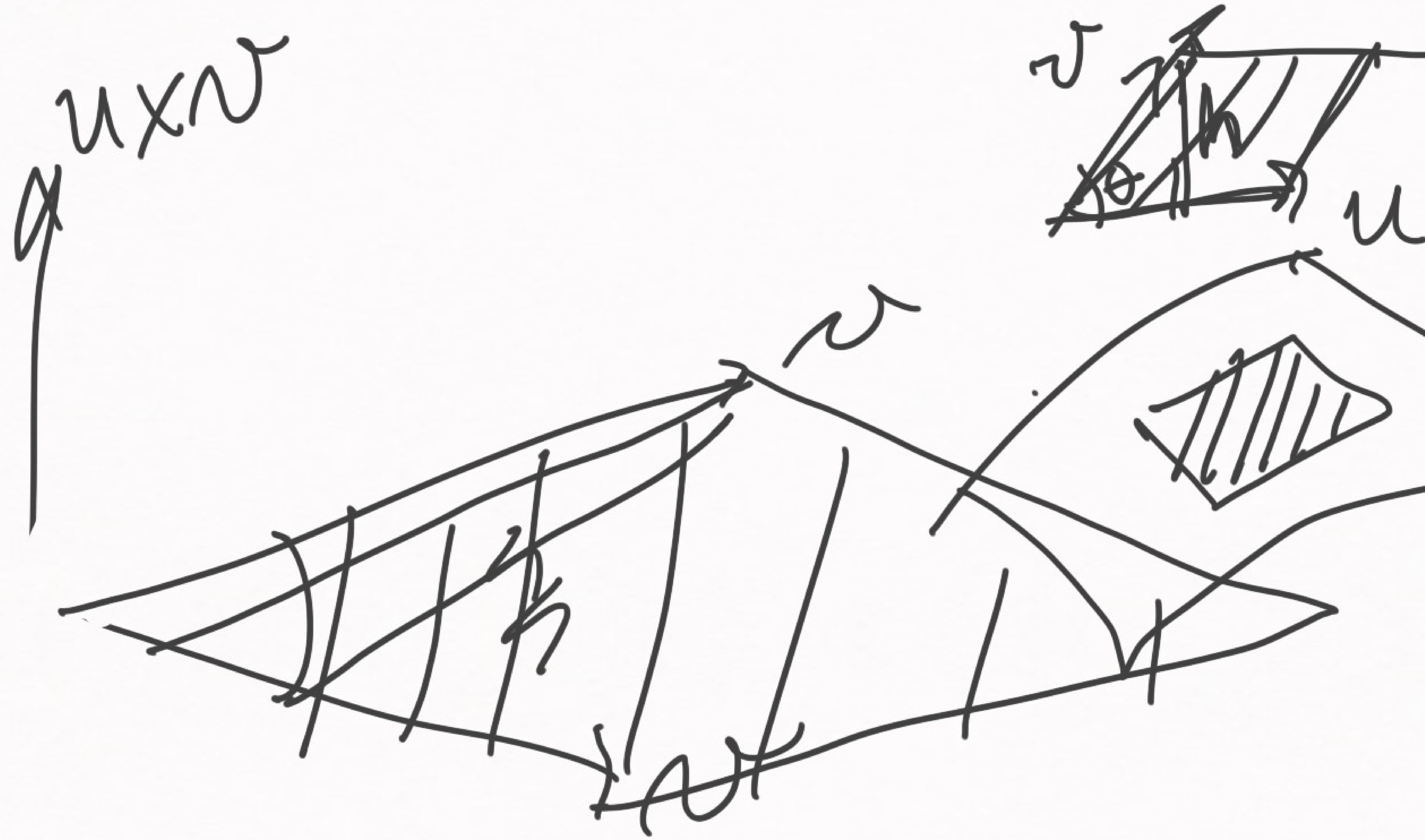
2_ direção

3_ sentido

1 —

$u \times v$ seja ortogonal a u e ortogonal a v





Area = $\|v\|\, h$

$\|u\| \operatorname{sen} \theta$

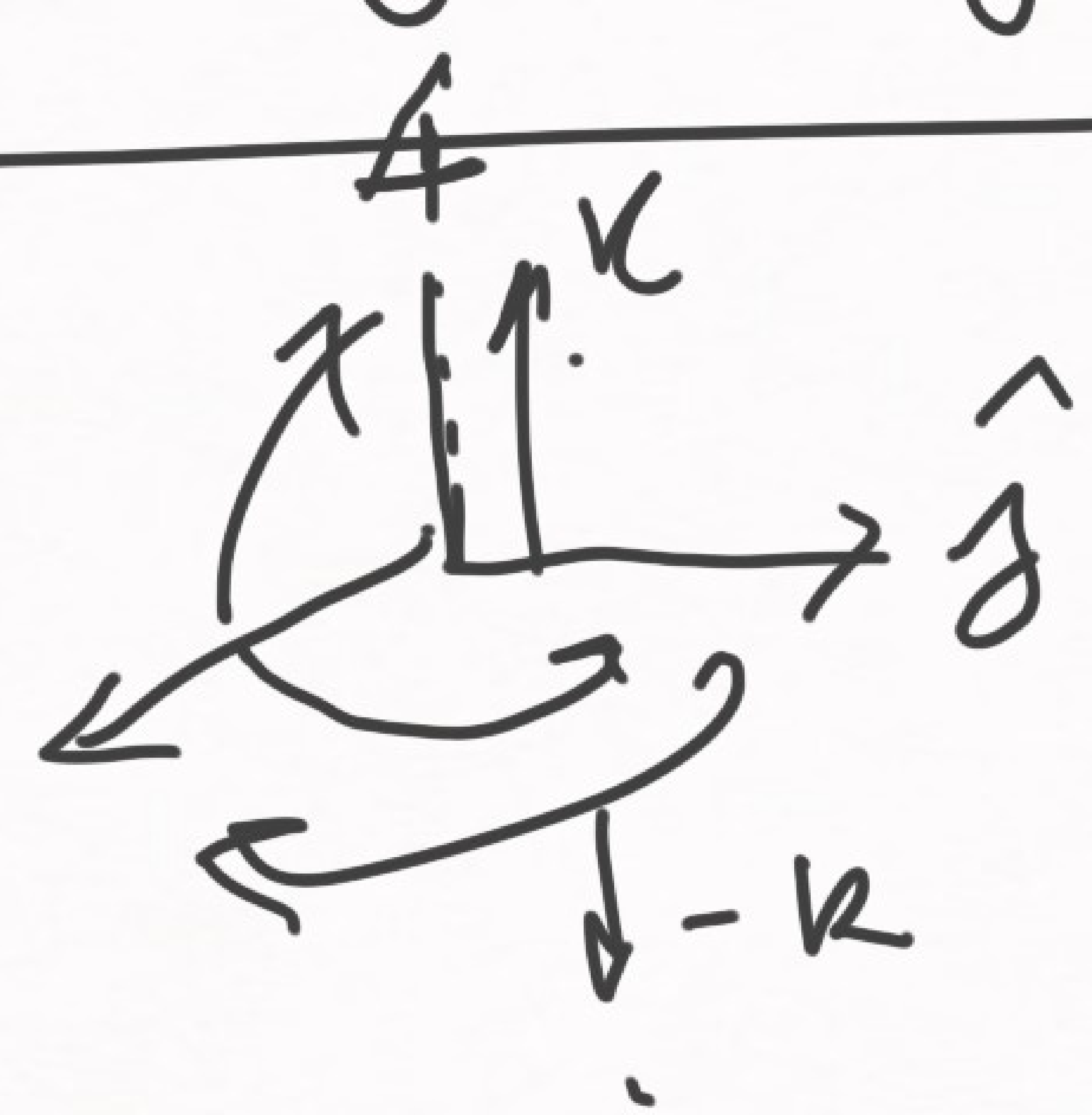
2- $\|u \times v\|$ = Area paralelogramo

$= \|u\| \|v\| \operatorname{sen} \theta$

Sentido Regra mão direita

$$u = u_1\hat{i} + u_2\hat{j} + u_3\hat{k}$$
$$v = v_1\hat{i} + v_2\hat{j} + v_3\hat{k}$$

$$\hat{i}\times\hat{i} = 0 \qquad \hat{j}\times\hat{j} = 0 \qquad \hat{k}\times\hat{i} = \hat{j}$$
$$\hat{i}\times\hat{j} = k \qquad \hat{j}\times\hat{i} = -\hat{k} \qquad \hat{k}\times\hat{j} = -\hat{i}$$
$$\hat{i}\times\hat{k} = -j \qquad \hat{j}\times k = \hat{i} \qquad \hat{k}\times\hat{k} = 0$$



"<u>determinante</u>"

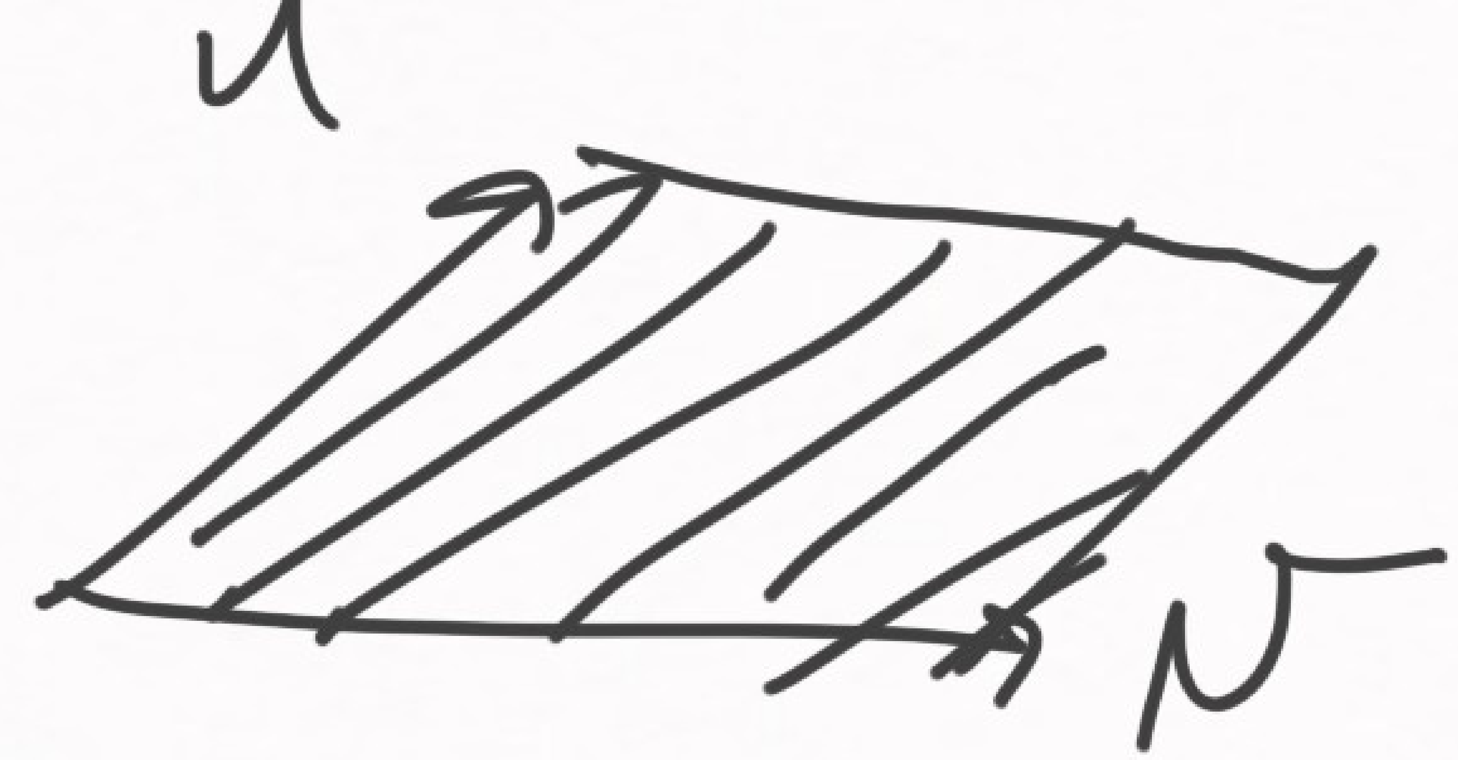
$$u \times v = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ u_1 & u_2 & u_3 \\ v_1 & v_2 & v_3 \end{vmatrix} =$$

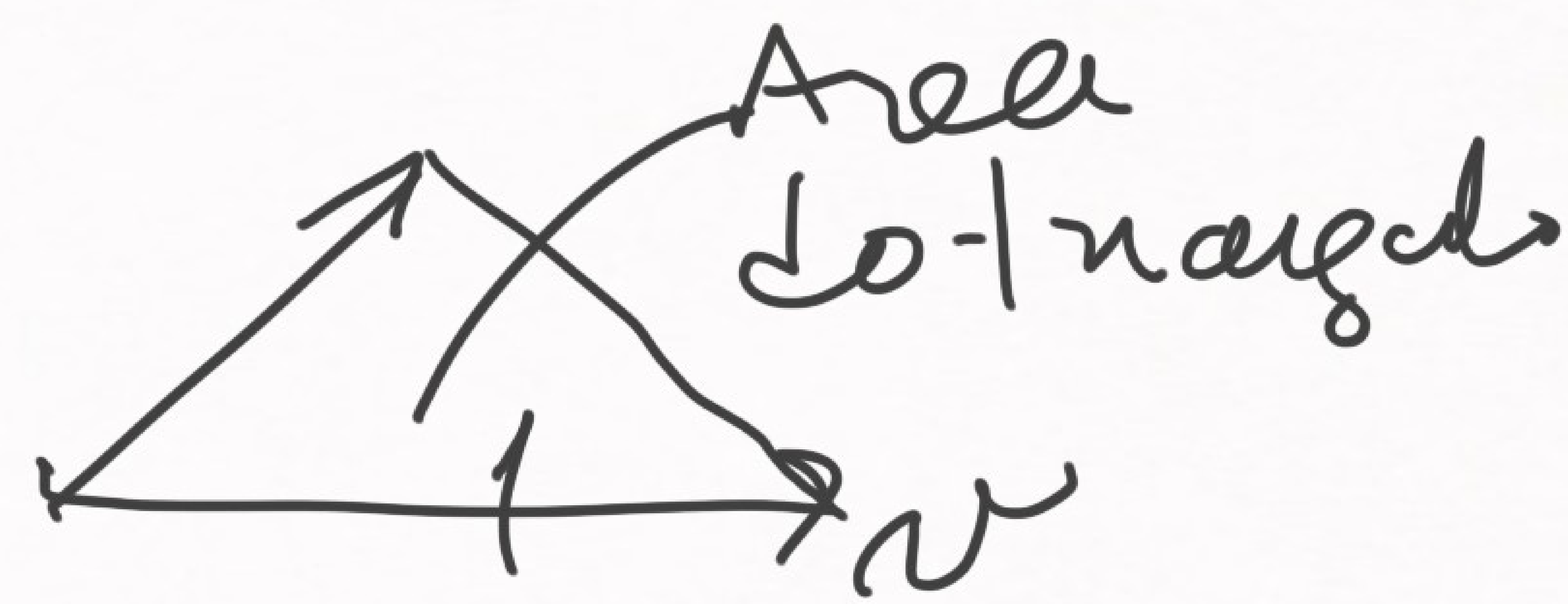
$$= \begin{vmatrix} u_2 & u_3 \\ v_2 & v_3 \end{vmatrix} \hat{i} - \begin{vmatrix} u_1 & u_3 \\ v_1 & v_3 \end{vmatrix} \hat{j} + \begin{vmatrix} u_1 & u_2 \\ v_1 & v_2 \end{vmatrix} \hat{k}$$

Caracterização do Produto vetorial

$$u \times v = 0 \Longleftrightarrow u \parallel v$$

1) $\|u \times v\| =$ Área paralelogramo dado por u e v





$$\frac{\|u \times v\|}{2}$$

Produto misto:

$$(v \times u) \cdot w = \langle v \times u, w \rangle$$

memória

$$\det \begin{vmatrix} w_1 & w_2 & w_3 \\ v_1 & v_2 & v_3 \\ u_1 & u_2 & u_3 \end{vmatrix} = w_1 \det \begin{vmatrix} v_2 & v_3 \\ u_2 & u_3 \end{vmatrix} - w_2 \det \begin{vmatrix} v_1 & v_3 \\ u_1 & u_3 \end{vmatrix} + w_3 \det \begin{vmatrix} v_1 & v_2 \\ u_1 & u_2 \end{vmatrix}$$

$\det \quad \hat{i} \quad \hat{j} \quad k$

$(v \times u)_i$

$i=1$

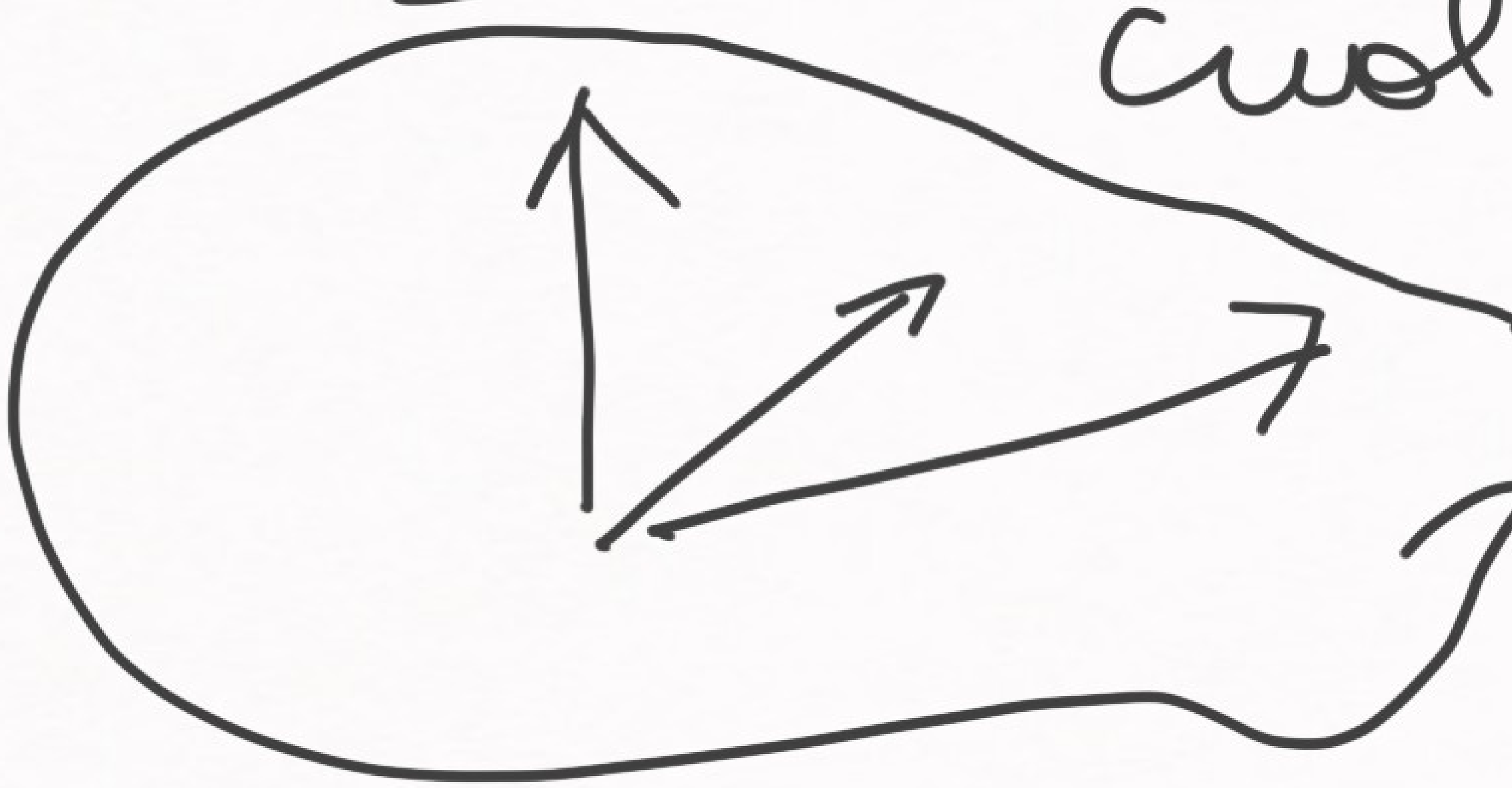
$i=2$

$i=3$

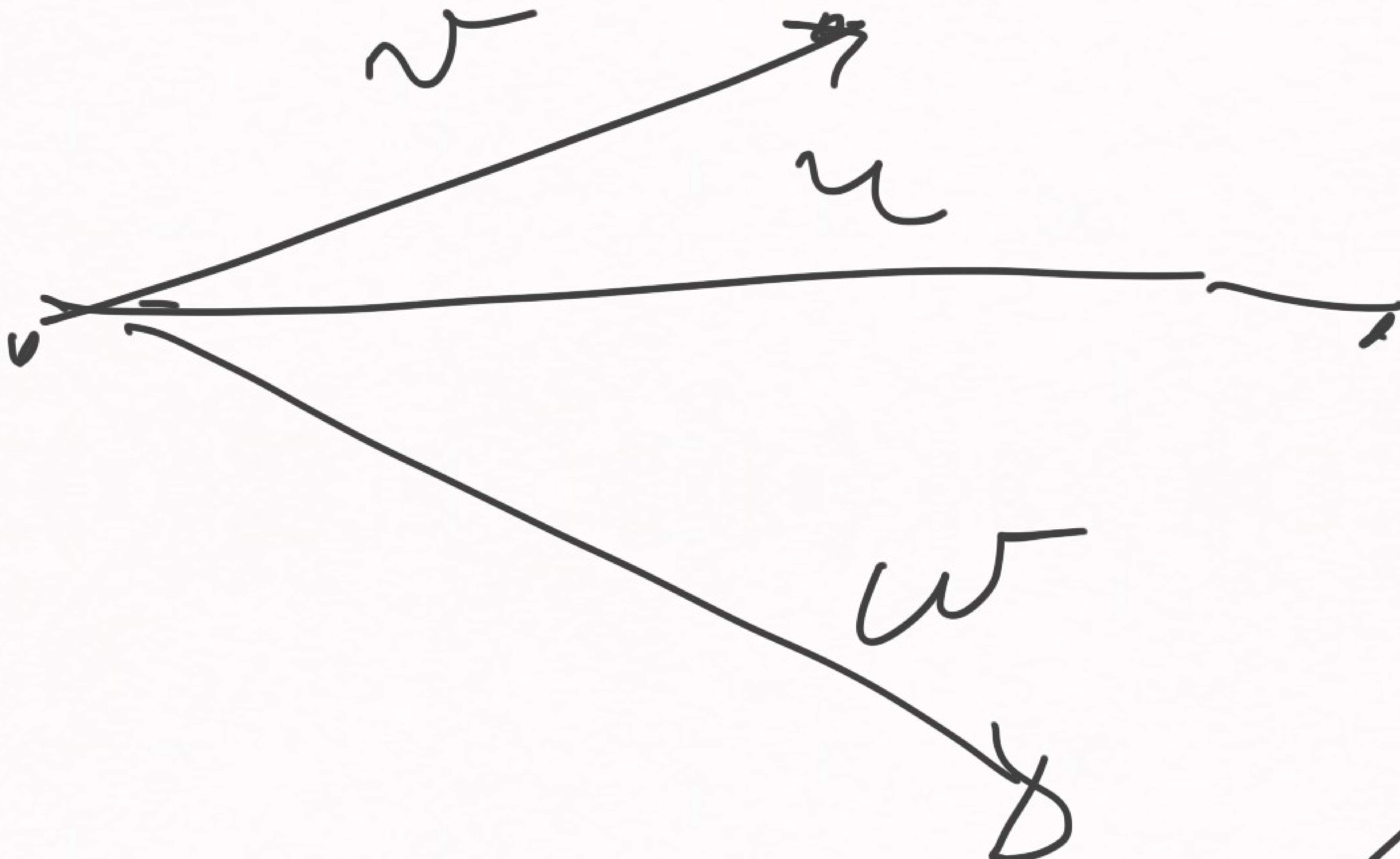
Caracterização que é importante

$\underbrace{u, v, w}_{\text{vetores}}$ são coplanares se

$$\boxed{\underbrace{(u \times v) \cdot w}_{\text{[illegible] produto misto}} = 0}$$



coplanares



$(v \times u) \cdot w = 0$ → são 4 pontos coplanares

$\neq 0$ → não